**Word Problems – the healing touch**

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It was a mathematics class in a government school in a busy market locality - typical for such a locality in any part of the country. The class was full of benches with about 40 children spread over them.

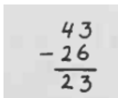
But there was a difference. The teacher had drawn some mangoes on the blackboard and was having a conversation about mangoes. Neelam, one of the children was explaining about the different types of mangoes. It was obvious that she had participated in sorting the mangoes at home to help her father prepare the *redi* for going into the market. She knew of the colour of *sindoori,* the sweetness of *dussheri* and that one had to wait a little longer for *chausa* which was her favourite. Her classmates were totally enthralled and the teacher had also a few things to learn from those three minutes of exposition.

Then the teacher introduced the character Mohan who went to Azadpur mandi to buy the fruits early in the morning to put up the redi for selling. She asked them how many mangoes would be there to sell today if Mohan had got 35 *dussheri* and 15 *safeda*? Immediately the class was full of raised hands with eager faces ready to answer. Teacher noted the energy and enthusiasm in the class. She asked one of the children how many mangoes would be there and also how she had found the answer so quickly. The child answered to say that she had put 5 into 35 and it became 40 and then put another ten into it and it became 50! There was another child who said she put 10 first and it became 45 and then another 5 and it became 50!

Then she decided to try one which was more challenging.

She said, "Umm… you know in the evening there were only three mangoes left. So how many mangoes do you think he sold?" Again there were many hands that went up and children could easily say,"47!" She was surprised to see that the class did not pause to ask whether they should add or subtract. Indeed it was as if they could practically see it in front of their eyes….the 47 so close … just 3 behind 50!

The teacher looked at the happy faces of the children and heaved a sigh. A feeling quietness spread through her and her eyes almost welled over. She remembered how her class used to be and the difference was palpable. She remembered the deadpan faces earlier as the maths classes started. In spite of her best efforts she had found it difficult to deal with it. What was most disturbing was the look of anxiety, with the fear of not being able to get the right answer. Through many years of drill most of the children had learnt to write the numbers evenly one below the others and get the correct answer, at least for addition. But the procedure for subtraction was still a stumbling block.




Now with new insights she thought to herself, "How could I have earlier expected a seven year to understand that a smaller number should be taken away from a bigger number only if the bigger number is written above? … In any case the children were only manipulating digits and not dealing with numbers. And it was not a question of logic but just drill."

The teacher suddenly snapped out of her train of thoughts and saw that the class was getting busy with other things. She decided to try something more difficult. She asked, "What if Mohan had 46 *dussehri* and 28 *sindoori*?" Now there was no immediate show of hands. But there were thinking faces trying to grapple with the numbers. She decided to change the question. "Now tell me will it be more than 60 mangoes or less than 60?" Suddenly the class was energised again. She then decided to work on the blackboard collectively to work out the answer to the question. She also knew that the children needed more counting experiences to deal with all kind of numbers and also support with the notation needed to keep track of the thinking process. But the healing had started and she was now confident of the future.

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The above scene could have been anywhere – a government school or a private school in India and also in most parts of the world. Because it is only lately that the awareness that the existing practices in mathematics are harming children has started spreading.

A study which was published in 1994 by Constance Kamii of the University of Alabama, U.S.A, showed that learning the standard algorithm severely compromised the ability of children to deal with numbers in a meaningful manner. She had been supporting teachers who delayed the introduction of the algorithms in her school and who instead gave the space to children to solve addition and subtraction using methods comfortable to them. Kamii found that the percentage of children who could correctly solve an addition problem mentally in algorithm based classrooms was significantly less than the percentage of children who had not been taught the algorithm.

The strategies of the `No algorithm' group had involved dealing with numbers rather than manipulating digits. The answers which this group gave were also much more reasonable, even when not exactly correct, than the answers of the group who had been taught the standard algorithm. So both in terms of the percentage of correct answers and the range of answers, the `No algorithm' group did much better. This was indeed an eye opener for Kamii. She wrote in introduction to the second edition of her book on arithmetic, "We have felt the need to revise the first edition of this book since we came aware of the harmful effects of teaching "carrying" and "borrowing" to second graders. When we wrote the first edition of this book in 1988, we had no idea that this teaching was harmful to children. We thought that our students, who invented their own procedures of multidigit addition and subtraction, were just doing better than those receiving traditional instruction" (Kamii, 2004, p vii)

**Table 1: Answers to the question (7+52+186) / (6+53+185) done mentally**

| Class | % Correct answer (245/ 244) | | % of reasonable answers (includes correct answer) (255-234) | | Answers more than the reasonable range | |
|---|---|---|---|---|---|---|
| | Algorithm sections | `No Algorithm' section | Algorithm sections | `No Algorithm' section | Algorithm sections | `No Algorithm' section |
| II | 12% | 45% | 12% | 70% | 9308, 1000, 989, 986, 938, 906, 838, 295 | 617 (was taught algorithm by mother at home) |
| III | 32% | 50% | 47% | 80% | 838, 768, 533, 800+ 38, 800 444, 344 | 284 |
| | 20% | | 40% | | | |
| IV | 30% | No section | 35% | No section | 10099, 1300, 1215,848, 844, 838, 835, 814, 791, 783, 783, 745, 744, 738, | No section |
| | 24% | | 38% | | | |
| | 19% | | 19% | | | |
| | 17% | | 33% | | 721, 718,715, 713+8, 713, 445,274 | |

(Calculated from the three Tables 17.1, 17.2 & 17.3 in Kamii & Dominick, 1998, pp 134-136)

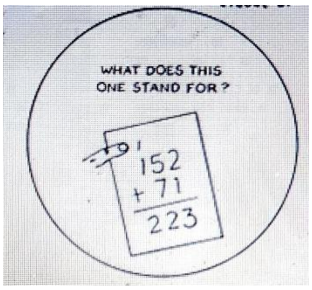
Table 1 shows how the children who had been taught to add and subtract using the standard algorithm using `carry-over' and `borrowing' were in fact handicapped. The other group of children had not been taught to do the algorithm and had been doing addition and subtraction using methods which made sense to them. According to Kamii, when asked to add mentally 7, 52 and 186, most children of `No Algorithm' group added by saying, ` 180 and 50 is 230' and then went on to add the ones. As can be seen from Table 1, the children who were not being taught the algorithm in both the grades 2 and 3 did much better. By grade 4, all the section teachers were teaching the algorithm. We can see that even by Grade 4, these children's ability to deal with the problem given did not match up to the ability of the `No Algorithm' sections of Classes 2 and 3.

Kamii also noticed some children in Class 4 were giving what we would consider as strange answers. Instead giving a number, as the sum of adding the three numbers, children had just given a string of three digits separated by commas. The answers they gave were as the following: 4,4,4,; 1,3,2; 8,3,7; 8,1,7, 8,3,8; 4,3,2; 4,3,2. (Kamii & Dominick, 1998, p 134). These can be seen to indicate that children were more and more thinking in terms of digits and not in terms of numbers. In the absence of the written vertical format to solve the problem, they resorted to just writing down the digits to indicate their thinking!

*Kamii had this to say of her experience*
"A characteristic of this class was the children's emotional flatness, and the fact that no one seemed to feel anything wrong with answers in the 800s, 900s and beyond. The children seemed to be functioning like machines, without any intuition for number."

Studies also showed that the problem was not just about the difficulty children had in dealing with orally given sums. Children who were being taught the algorithm also had difficulty in making sense of the standard algorithm which they were being taught from Class 1.

In fact by mid nineteen eighties many researchers had started noticing the impact the standard algorithm was having on the thinking processes of children. Ed Labinowicz noted that in Class 3 children who were adept at following the rules of the standard algorithm were not able to answer the meaning of the `1' which was carried over. He found that among the 24 third grade students he studied, 19 solved the written sum correctly using the standard algorithm, but only one among them was able to explain their procedures spontaneously using place value ideas. (Labinowicz, 1985, p 216-17). Even with follow-up questions only half the children were able to give any kind of place value connection for the procedure they followed.




In our own experience we found that even class 5 children find it difficult to explain what the `1' stands for. Carry-over from ones place to the tens place was still understandable to some children, but understanding that the `1' which comes from 12 got by adding 5 and 7 is a 100 is something that was beyond them.

**Making the algorithm work**
Faced with the difficulties children were facing with the algorithm, many attempts were made to use activities with concrete materials to teach the algorithm. The idea was to `teach' place value to children so that they can do the digit based algorithm with understanding. These attempts basically assumed that if the mathematical relationships could be embodied in materials, then children would be able to understand the logic of the algorithm. Mapping Activities between concrete materials such as Dienes blocks and the algorithmic procedures were undertaken widely. The impacts of such

mapping activities were also studied by researchers in U.S.A. and elsewhere. But these studies at best gave mixed results about the usefulness and indicated the problems children faced.

The reason that the base ten blocks did not help children is understandable from the perspective of developmental psychology of learning. An adult who knows the mathematical relationship can `see' the relationship between tens and ones in the material, but a child who does not know the relationship cannot `see' the relationship of tens and ones in the small cubes and rods of the Dienes block set.


Cognitive psychologists Lauren Resnick and S. Omanson conducted a study involving detailed instruction for mapping between the written procedure and the use of materials. They concluded that this approach did not work and that an approach of a different order would be required in order to help children learn the algorithm. Only 2 children out of 9 children who followed the instruction could do subtraction correctly in the test given immediately after the instruction. (quoed in Gravemeijer, 1994, p 62). In fact counting 10 and then seeing it as a ten requires a higher level of abstraction in thinking, it requires the ability to see the ten ones as ones and as a ten simultaneously. Concrete materials cannot be used to make children `see' mathematical relationships but they can be used to have `grounded conversations' and communication between the children and the teacher (Thompson, 1994). This same idea is also echoed by a recent mainstream review of the studies which points out that the studies using concrete materials produced mixed results because the crucial factor of the psychology of the child, ` the internal factor' was ignored. (Baroody, 2017, p 45)

**Digit manipulation – a solution and a danger**

If children are having so much difficulty with understanding the algorithm, then the question naturally emerges; why is it being taught? The standard algorithm emerged at a particular historical period as a powerful tool to find answers quickly. It's strength lies in the fact that it is based on the manipulation of a limited number of elements, the digits. This is true for even large numbers. With sufficient drilling of the multiplication facts (Tables) and earlier the addition and subtraction facts, correct answers can be expected with very little load on working memory. But when children are asked about why the answer is what it is, then these children do not have an answer as Kamii, Labinowicz, Thompson and many others have found out.

Even though the standard algorithm might be helping children to add larger numbers, the danger that it might disconnect them from their own thinking processes exists, because it goes against the spontaneous intuitive sense to deal with the larger numbers first. For example when adding 487and 345, children and even adults would first add 300 and 400 to say 700 and then go further. In this case, one is dealing with numbers and not with just digits.

> When young children are taught to do addition by starting from ones rather than from tens, they are being told to suspend what makes sense to them and asked to do the opposite. We can well imagine what such a process could have on the child.

In the early nineties, researchers from Portland State University in the USA (Narode et. al. 1993) studied the impact of teaching the algorithm on the thinking process of children. Interviews were conducted with children about solving simple word problems involving two digit addition and subtraction before and after they were taught the standard algorithm. All the nine students of Grade 2 who were interviewed before being taught the algorithm used front-end methods (adding tens first and then adding ones) for doing the addition. Six of the nine students successfully solved the problem using their own mental methods. Children showed different flexible methods.

The researchers report the case of Jamie who added 19 and 26 mentally in the beginning of Class 2. "I know I have 30 because I have a group of ten and two more tens. Then if I take a 1 from the 6 and give it to the 9, I'll have another group of 10. That leaves five left, so the answer is 45."

That was in November. By May, the following year the same Jamie emphatically stated, "No you never add the tens first" (Narode et. al. p 259); a clear indication that Jamie (and countless others like her) had been forced to suspend her own sense-making intuitions. When children grow up like this, they tend to believe that mathematics is simply a question of following rules and procedures.

**The way ahead**

The way ahead would emerge from our own sensitivity to how children think and feel. We need to recognise that children are natural problem solvers – they solve problems using whatever means are available to them, whether by pulling a rug or by using a step to climb on, to reach what they want. Their repertoire increases as they participate in problem solving activities along with adults especially with the support of language in communication. In the current school context, word problems can keep alive the thinking ability of children. When the context is something which makes sense to them and they are able to visualise the situation, then they can use their number sense to solve them. Then children would not ask,"Ma'm should I add or subtract". After all many problems can be solved using either addition or subtraction.

What is crucial is giving children the space to think and solve problems and supporting their further development with appropriately designed activities. Classroom conversations to share ideas and to think through problem situations support the ability of children to deal with more and more complex problems. As the number sense of children develops, first through various counting activities to ascertain quantity and then through other activities, the range of word problems increases.

Children need to first get the space to count in meaningful ways, to play with numbers, to do naturally counting–on to find out what happens something is added or combined and many such activities before they are asked to split numbers and to learn place value. Giving children opportunities to deal with numbers rather than with just digits, help to develop a strong base for number sense. Tools appropriate to show the thinking processes of children can be introduced into the classrooms which also help to increase the ability of children to keep a track of their thinking processes.

The experience of Jodo Gyan in working with children has shown that when children are given the space to think, they are able to solve word problems of different types using strategies and methods which make sense to them. Without the pressure on somehow reaching the correct answer, there is a natural space for developing their confidence. Word problems pitched at their level can indeed be the healing touch.

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