What makes mathematics difficult for children?

"Is 18+4 an easy or difficult question?"

It was a group of about 40 teachers of young children and all of them were unanimous to say that it is a difficult question.

They had so many things to say about the difficulties children would have… Some said, "Writing itself would be difficult". They won't write properly. They will write 4 below 1!

$$\begin{array}{r} 18 \\ +4 \\ \hline \end{array}$$

Another said, "Even if they write properly, they still make mistakes!" They don't know how to do carry-over"

$$\begin{array}{r} 18 \\ +\ \ 4 \\ \hline 112 \end{array}$$

"Yes, they need more practice!"

I thought it over and asked them, What if you had asked them, 'You made a mala of 18 beads and then you added 4 more beads. Then how many beads will be there in your mala?'

Teachers were unanimous in saying, "Oh! Then they will be able to do it!"

This small episode captures in a nutshell something that plagues our education. Not just in mathematics but in almost, nay, in all subjects in education. We seldom start from what children can do but usually start from where we think they need to reach. Mathematics is a very good example of this approach. Once children can count comfortably, they use this capability to solve problems. The first step is to count ahead when they are confronted with a problem where the sum has to be found. Counting ahead is done not only to find the sum but also in situations where the difference is to be found.

Appu's parents decided to celebrate nani's birthday with all the neighbours. They decided to gift a mango to all the guests to take home. About 50 guests were expected. When they checked in the morning they found that only 46 mangoes were ripe. So how many mangoes need to be got?

In this case, the calculation that would be done would be to make a mental jump forward and say `4'. In school this would be a subtraction sum in which children would be getting into knots.

Giving children the opportunity to think in the context is what a modern education needs to be doing. Today the requirement is not to have people who can add up a string of digits to find the right answer, but of people who can model a situation and find the correct solution method.

Research in mathematics education is a fairly young discipline. In the early years the focus of the research was to understand the reason why children made mistakes in the standard method they were being taught at school, as in the vignette with the teachers above. This research opened the eyes to the `bugs' which had crept into the school taught procedures.

One response to this discovery was to make children understand the logic of the procedures of the standard school algorithms. Another response was to look at children, observe children's thinking in situations that involved finding sums and differences so that we as adults could understand how children think and respond and only then devise methods by which their mathematical thinking could develop further. There was yet another response or lack of a response, in which more and more drilling and repetition was done to make children do the procedures exactly as taught.

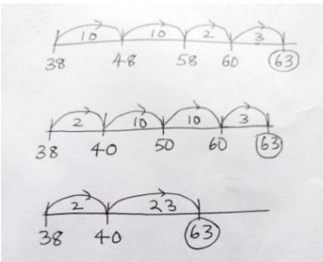
The first response was dominant in the eighties and nineties with the focus on `teaching' children place-value so that they could do the standard algorithm with understanding. Whether the children were psychologically ready to understand a concept such as place-value was not the focus of these researches at that time.

The second response in research which looked at how children do problem-solving in contexts rather than just teach the digit-based algorithm, is the other stream of research which is gaining more attention in the last thirty years. It has also being reflected in the curriculums in countries such as New Zealand and has informed international assessment programs such as the OECD sponsored PISA. Wider understanding in society about the methods and grounding of the implementation processes is the current challenge.

But before we look at possible alternatives emerging to the standard procedures taught in schools, let us have a brief look at what happens in the process of doing addition in the old method.

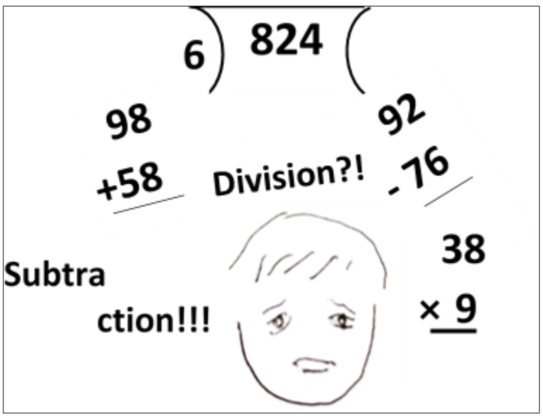
Doing the standard digit-based Algorithm

Let us first consider the situation where a child with number sense has to add the 38 and 25.



When doing mentally she might add 30 and 20 to say 50 and then add the 8 and 5 to make 13 and then add the two. It is a natural tendency to add the bigger numbers first. There are other methods also that children use. One child might take 2 from 25 to make 38 into 40 and then add the rest. Another might make jumps of ten or more. But in all these cases, children are thinking about *numbers and not about digits*. They are adding 30 and 20 and not 3 and 2. Their thoughts have a sense of quantity or magnitude attached to it. (Pic to be changed)

But with the standard algorithm the child is operating with just digits. In this case, she is expected to follow the procedure step by step and start by adding the ones rather than the tens. She has to first add the digits 8 and 5. She can do this only by suspending her intuitive reaction to add the big numbers.



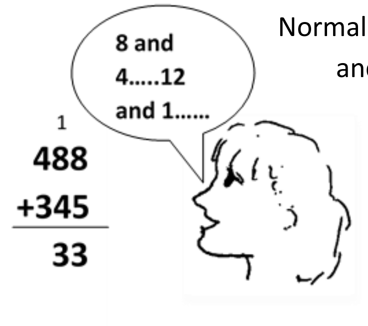
In fact in this process she is being made not to trust what makes sense to her. This can have major consequences for her self-confidence and ability to do problem solving using methods which make sense.

Constance Kamii, a researcher from the University of Alabama, wrote that what struck her about the class of children who had been taught the standard algorithm was `the children's emotional flatness, and the fact that no one seemed to feel anything wrong' with their answers which were completely unreasonable for the question asked.' [1]

It is as if children are caught in a web of calculations with no connection to quantity or meaning.

What is in the standard digit-based algorithm?

Let us look at what is needed for children to do the standard algorithm and still make sense of it. Normally when children add 488 and 345, they are thinking about 8 and 4 and not 80 and 40 and same for 400 and 300. They are just adding 4 and 3.



In order to add by simply adding the digits and still make sense of it, children need to understand 80 as 8 tens or 8 times ten which involves understanding multiplication and the multiplicative basis of place value. Without that, it becomes just an exercise in digit manipulation built on the vestiges of underdeveloped number sense. In this case, even children who are able to find the correct answer are not able to think whether adding 488 and 345 would be more than thousand or less than thousand. This making sense emerges through participation in many carefully-designed activities that are meaningful for children.

When children get sufficient opportunity to count one by one, they develop an understanding of `ten' as a composite unit. Or in other words they are able to think of a `ten' as a unit as well as being composed of ten sub-units. The research by Paul Cobb[2] showed how this process of `ten making' is affected by the early introduction of the standard algorithm. By the end of eighties series of research studies by Kamii, Labinowicz and others showed[3] the many problems associated with the introduction of the standard digit-based algorithm in the early years.

Another path?

But there have been other approaches that have considered building number based methods by looking at children and their process of building meaning. There are organisations and institutions including in our country that have been working with children to develop/introduce in practice number sense based methods for mathematical operations.

These methods introduce written methods in tandem with the developing number sense of children. The focus is on the evolution of the mathematical thinking processes of children and number-based methods get priority. In this process problem solving with context-based word problems lead the way before bare number calculations are introduced. This early connect with everyday life means that children would be able to use their knowledge later for problem solving in different real-life contexts.

[1] Kamii, C., & A. Dominick (1998). The Harmful Effects of Algorithms in Grades 1-4. In L. J. Morrow & Margaret J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics* (1998 NCTM Yearbook). Reston, VA: National Council of Teachers of Mathematics.

[2] Cobb, P., & Wheatley, G. (1988) Children's Initial Understanding of Ten. *Focus on Learning Problems in Mathematics,* 10 (3), 1-28.

[3] See Menon, U. (2018), `Word Problems –the healing touch' available at http: for the references to these studies.

Digits and digit-based algorithms

One might wonder what could be the reason behind the widespread teaching of the digit-based algorithm. This could be related to the need in earlier centuries to do long calculations as part of accounting activity. Today we use the calculator or spreadsheet to do our accounting. What we would need today would be the ability to estimate whether the calculator has given the correct answer and be able to design new processes for calculators. All these need number sense, the ability to see number relationships with a sense of the quantity or magnitude involved.

When the digit-based algorithm is introduced early, children tend to see number as a string of digits and the sense of number connected to magnitude gets lost.

At this stage it is useful to remember that the notation for number that we have all studied in school is only one form of representation of number. Number as a concept can be represented in many ways. The mathematical properties of number remain the same whichever representation is used for number. 17 Is an odd number and a prime number in our decimal base system or in binary system or even in a system without a base, like in Egyptian hieroglyphics based representation of numbers.

The way forward

One can say, that in the last few decades the walls created by digit based methods are already crumbling and it is not so universal as it had been. Thus for example, the current New Zealand mathematics curriculum for Level 3 expects that children should be able to use whole number strategies to solve word problems involving partitioning or combining of whole numbers. For eg. In a problem involving 26 people getting down from a bus of 53 people the strategies could involve methods such as 53-20= 33; 33- 6 = 27; or adding 20 to 26 and then adding 7 to reach 53. As a note it adds that `If the student uses a written algorithm to solve the problem, they must explain the place value partitioning involved', apparently as a way to deal with old digit based methods. [4]

We can wonder why this system of calculation which creates fear of mathematics continues. One can only say that this digit based calculation is so embedded in our current mass education system with linkages across different institutions, involving class examinations, marks and grading, syllabus setting, entrance exams, teacher education and so on. It is also being strengthened currently through large intervention programs that expect quick solutions. This system is so interlinked that it is difficult for change to come and be sustained. The change will have to come from outside, from the ground and may be informed parents can play an important role. When there is a groundswell for working for an education that considers the lived reality of the classroom, then lives of our children would change and their fears would disappear.

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30 -06-2019

[4] Ministry of Education (2009).The New Zealand Curriculum, Mathematics Standard , Years 1-8, p31. Available at https://nzcurriculum.tki.org.nz/content/download/3166/47235/file/Maths_Standards_amended_vs3.pdf